

Consider the sequence $5, -45, 405, -3645, 32805, \dots$.

SCORE: ____ / 13 PTS

$\times(-9) \times(-9) \times(-9) \times(-9)$ GEOMETRIC

[a] Find the formula for the n^{th} term of the sequence.

$$a_n = \underline{5(-9)^{n-1}} \quad (2)$$

[b] Find the formula for the sum of the first n terms of the corresponding series $5 - 45 + 405 - 3645 + 32805 - \dots$.

$$S_n = \frac{5(1-(-9)^n)}{1-(-9)} = \underline{\frac{1}{2}(1-(-9)^n)} \quad (2)$$

[c] Prove that your formula in [b] is correct using mathematical induction.

BASIS STEP: $S_1 = 5$

$$\textcircled{1} \quad \frac{1}{2}(1-(-9)^1) = \frac{1}{2}(10) = 5 \checkmark$$

INDUCTIVE STEP: ASSUME $5 - 45 + 405 - \dots + 5(-9)^{k-1} = \frac{1}{2}(1-(-9)^k)$ $\textcircled{1}$

$\textcircled{1}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$\textcircled{1} \quad \underline{5 - 45 + 405 - \dots + 5(-9)^{k-1} + 5(-9)^k}$$

$$\textcircled{1} \quad \underline{\frac{1}{2}(1-(-9)^k) + 5(-9)^k}$$

$$\textcircled{1} \quad \underline{\frac{1}{2}(1-(-9)^k + 10(-9)^k)}$$

$$\textcircled{1} \quad \underline{\frac{1}{2}(1+9(-9)^k)}$$

$$= \frac{1}{2}(1-(-9)(-9)^k)$$

$$\textcircled{1} \quad \underline{\frac{1}{2}(1-(-9)^{k+1})}$$

$\textcircled{1}$ BY MI, $5 - 45 + 405 - \dots + 5(-9)^{n-1} = \frac{1}{2}(1-(-9)^n)$
FOR ALL INTEGERS $n \geq 1$

FJ & GJ are friends, so when FJ opened a Twitter account, GJ started following it. The day the account was opened, **SCORE: ____ / 7 PTS** GJ received 11 tweets from FJ. Every day after that, GJ received 6 more tweets from FJ than he had received the previous day. One day, GJ noticed that, for the first time ever, he had received more than 200 tweets from FJ that day. As a result, GJ stopped following FJ's Twitter account, and recommended that FJ start attending Tweepheads Anonymous. **ARITHMETIC $d=6$**

[a] How many days had FJ's Twitter account been open when GJ stopped following?

$$\begin{aligned} \textcircled{2} \quad & 11 + 6(n-1) > 200 \quad \textcircled{1} \\ & 6(n-1) > 189 \\ & n-1 > 31\frac{1}{2} \\ & n > 32\frac{1}{2} \\ & \underline{n = 33} \quad \textcircled{1} \end{aligned}$$

GJ STOPPED FOLLOWING
ON THE 33RD DAY

[b] How many tweets had GJ received altogether by that time?

$$\begin{aligned} S_{33} &= \frac{33}{2} (2(11) + 6(33-1)) \quad \textcircled{2} \\ &= \underline{3531} \text{ TWEETS} \quad \textcircled{1} \end{aligned}$$

Expand and simplify $(2t^3 - \sqrt{t})^4$. **The coefficients in your final answer must be completely simplified.**

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$$\begin{aligned} & \underline{(2t^3)^4} + \underline{4(2t^3)^3(-\sqrt{t})} + \underline{6(2t^3)^2(-\sqrt{t})^2} + \underline{4(2t^3)(-\sqrt{t})^3} + \underline{(-\sqrt{t})^4} \\ &= \underline{16t^{12}} - \underline{32t^9\sqrt{t}} + \underline{24t^7} - \underline{8t^4\sqrt{t}} + \underline{t^2} \end{aligned}$$

$$\begin{array}{r} 1 \\ 1 \ 1 \\ 1 \ 2 \ 1 \\ 1 \ 3 \ 3 \ 1 \\ 1 \ 4 \ 6 \ 4 \ 1 \end{array}$$

$\frac{1}{2}$ POINT EACH

Find the coefficient of x^6 in the expansion of $(5x^2 - 3)^{43}$.

SCORE: ____ / 5 PTS

You may write your final answer in factored form, as shown in lecture.

Your final answer must **NOT** contain ! or $C(n, r)$ (or equivalent) notation.

NOTE: Do NOT use your calculator's ! nor $C(n, r)$ feature.

$$\binom{43}{r} (5x^2)^{43-r} (-3)^r = \binom{43}{r} 5^{43-r} (-3)^r x^{2(43-r)}$$

$$\begin{aligned} 2(43-r) &= 6 \\ r &= 40 \end{aligned}$$

$$\textcircled{1} \quad \underline{\binom{43}{40}} 5^{43-40} (-3)^{40} x^6$$

$$= \frac{43!}{40!3!} 5^3 3^{40} x^6$$

$$\textcircled{1} \quad \underline{\frac{43 \cdot 42 \cdot 41 \cdot 40!}{40! 3 \cdot 2 \cdot 1}} 5^3 3^{40} x^6 = \underline{43 \cdot 7 \cdot 41} \cdot \underline{5^3} \cdot \underline{3^{40}} x^6$$